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Reconstruction of globoidal cam follower motion curve based on B-spline

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ABSTRACT*: Due to the development of the CNC technology, manufacturing the complex surface is becoming more and more easier. Globoidal cam is an important part of intermittent motion mechanism, so nowadays machining the globoidal cam by CNC machine is a new research area. In general, the contour line of the globoidal cam can be described by the follower motion curve. But because of the property of the globoidal cam, its contour surface is non-development, so the follower motion curve cannot defined by mathematical expression. In order to solve this problem, this paper attempts to reconstruct the follower motion curve based on B-spline by the help of MATLAB. In this research, 3 common follower motion laws are discussed: modified constant velocity, modified trapezoid and modified sine. By observing the results, it can be said that reconstruction of the follower motion curve is similar to the original curve which defined by mathematical equation.*

Keywords *-B-spline, Follower motion curve, MATLAB, Reconstruction*

I. INTRODUCTION

In the current development of the mechanical products, there are two directions: one is large-size and standardized, one is small-size and versatile. So an increasing number of automatic machine play an important role in this development.

Due to the limitation of speed, reliability and price, the automatic machines with cam and its related mechanism combination possess a large market. Globoidal cam mechanism is one of the most commonly used cams. It has the characteristics of simple structure, reliable operation and high accuracy. The quality of the contour surface effects the working performance of the globoidal cam directly. But the contour surface of globoidal cam is complex and non-developable surface, so it's difficult to design and manufacture [1].

In recent development of the CNC machine, some researchers attempt to design globoidal cam by B-spline and manufacture it by B-spline interpolation. In this way, it can reduce the machining errors. In this paper, B-spline curve was used to fit the movement curve of the follower that can represent the contour line of the cam.

II. METHODS

In general, engineers design the globoidal cam by the handbook. According to the follower mechanism, they try to design the globoidal contour parameters that can achieve the desired law of motion. But the contour surface of globoidal cam is non-developable curved surface, it cannot be described by mathematical formulation, hence this paper try to fit the contour surface by B-spline.

2.1 Follower motion law of the globoidal cam

The basic follower motion law of the globoidal cam are: constant velocity curve, constant acceleration curve and harmonic curve [2]. But they are so simple that may be cause vibration and shock. So in real situation, in order to exploit its strong points and avoid exposing its weaknesses, it discusses the combined motion law here [3, 4].

2.1.2 Modified constant velocity curve [5]

This curve is the combination of the constant velocity curve and harmonic curve. This curve is used for the situation which middle part has velocity that can be controlled in low level. This curve can be apply for middle speed and heavy load occasion.

In a cycle β (β is the turning angle of the cam when the follower passed its total angular displacement) of follower motion, the displacement equation can be described as:

$$y=\begin{cases}\frac{h}{5\pi+4}[2\pi\frac{\theta}{\beta}-\frac{1}{4}\sin(8\pi\frac{\theta}{\beta})] & 0\le\theta\le\frac{\beta}{16}\\ \frac{h}{5\pi+4}\{2+2\pi\frac{\theta}{\beta}-\frac{9}{4}\cos[\frac{8\pi}{3}(\frac{\theta}{\beta}-\frac{1}{16})]\} & \frac{\beta}{16}\le\theta\le\frac{\beta}{4}\\ \frac{h}{5\pi+4}(2-\frac{3\pi}{2}+8\pi\frac{\theta}{\beta}) & \frac{\beta}{4}\le\theta\le\frac{3\beta}{4}\\ \frac{h}{5\pi+4}\{(3\pi+2)+2\pi\frac{\theta}{\beta}+\frac{9}{4}\sin[\frac{8\pi}{3}(\frac{\theta}{\beta}-\frac{3}{4})]\} & \frac{3\beta}{4}\le\theta\le\frac{15\beta}{16}\\ \frac{h}{5\pi+4}\{(4+3\pi)+2\pi\frac{\theta}{\beta}+\frac{1}{4}\cos[8\pi(\frac{\theta}{\beta}-\frac{15}{16})]\} & \frac{15\beta}{16}\le\theta\le\beta\end{cases} \tag{1}$$

Equation1displacement equation of modified constant velocity curve (h is the maximum height of the follower motion, θ is the angle of cam rotation.)

Because the middle of acceleration curve is zero, so this curve is called modified constant velocity curve. The picture is shown below:

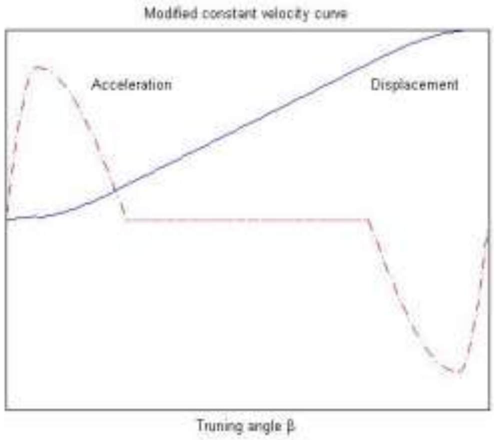

Fig 1modified constant velocity curve

2.1.2 Modified trapezoid curve

This curve is the combination of the constant acceleration curve and cycloidal curve. This curve has good continuity so that it can reduce the vibration. This curve can be apply for high speed and light load occasion.

In a half cycle $\frac{\beta}{2}$ of follower motion, the displacement equation can be described as:

$$y=\begin{cases}\frac{h}{2+\pi}(\frac{2\theta}{\beta}-\frac{1}{2\pi}\sin 4\pi\frac{\theta}{\beta}) & 0\le\theta\le\frac{\beta}{8}\\ \frac{h}{2+\pi}[\frac{1}{4}-\frac{1}{2\pi}+\frac{2}{\beta}(\theta-\frac{\beta}{8})+\frac{4\pi}{\beta^2}(\theta-\frac{\beta}{8})^2] & \frac{\beta}{8}\le\theta\le\frac{3}{8}\beta\\ \frac{h}{2+\pi}[-\frac{\pi}{2}+2(1+\pi)\frac{\theta}{\beta}+\frac{1}{2\pi}\sin 4\pi\frac{\theta-\frac{\beta}{2}}{\beta}] & \frac{3}{8}\beta\le\theta\le\frac{\beta}{2}\end{cases} \tag{2}$$

Equation2displacement equation of modified trapezoid curve

Because the shape of acceleration curve is trapezoid, so this curve is called modified trapezoid curve. The figure is shown below:

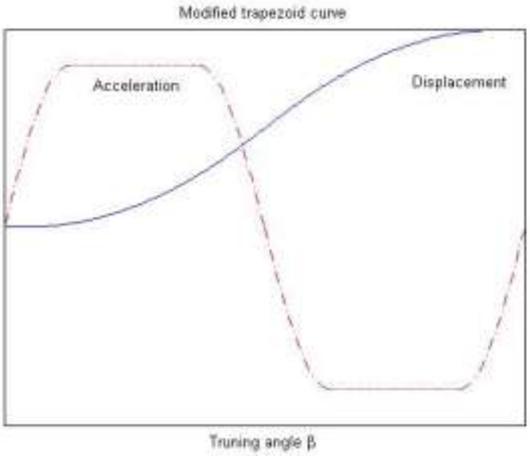

Fig 2 modified trapezoid curve

2.1.3 Modified sine curve

This curve is the combination of the cycloidal curve and harmonic curve. It has lower maximum speed and maximum acceleration than the modified trapezoid curve, so it can be used for high speed and middle load occasion. In indexing mechanism, modified sine curve is widely used because of its overall performance.

In a cycle β of follower motion, the displacement equation can be described: as:

$$y=\begin{cases} h[\frac{\pi}{4+\pi}\frac{\theta}{\beta}-\frac{1}{4(4+\pi)}\sin 4\pi\frac{\theta}{\beta}] & 0\le\theta\le\frac{\beta}{8} \\ h[\frac{2}{4+\pi}+\frac{\pi}{4+\pi}\frac{\theta}{\beta}-\frac{9}{4(4+\pi)}\sin(\frac{4\pi}{3}\frac{\theta}{\beta}+\frac{\pi}{3})] & \frac{\beta}{8}\le\theta\le\frac{7}{8}\beta \\ h[\frac{4}{4+\pi}+\frac{\pi}{4+\pi}\frac{\theta}{\beta}-\frac{1}{4(4+\pi)}\sin 4\pi\frac{\theta}{\beta}] & \frac{7}{8}\beta\le\theta\le\beta \end{cases} \quad (3)$$

Equation3displacement equation of modified sine curve

Because the shape of acceleration curve is sine, so this curve is called modified sine curve. The figure is shown below:

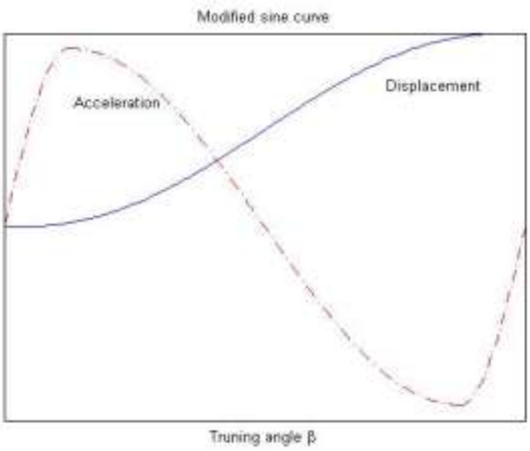

Fig 3modified sine curve

2.2 The use of B-spline

In this paper, B-spline is the tool for fitting the follower motion curve from the mathematical formula.

2.2.1 The development of B-spline

In the beginning, engineers and technicians designed the curve by thin and elastic metal strap that was called as “spline” [6]. Based on this actual working conditions, there was a mathematical function named cubic spline which has advantage of low degree, piecewise smoothness and easily computed. But as the complexity of the designed curve is higher and higher, the cubic spline was lack of intuition and flexibility, so French automotive design engineer Bezier introduced a new spline. This Bezier spline can change the shape of whole curve by adjusting the controlling point.

The shortcoming of the Bezier curve has poor ability for local controlling and curve modification. In order to reduce these disadvantages, people developed the B-spline based on Bezier curve from 1972 to 1974[7]. The B-spline not only keeps all the good bits of Bezier curve, but enhances the ability of local controlling.

2.2.2 The concept of B-spline

B-spline is consist of piecewise curve. In order to describe complex shape and local property, the single parametric curve can be defined as base function [8]:

$$P(u) = \sum_{i=0}^{n} d_i N_{i,k}(u) \tag{4}$$

d_i (i =0, 1…n) is the control points, $N_{i,k}(u)$ is the k degree piecewise polynomial that defined by non-decreasing knot vector parameters sequence $U : u_0 \le u_1 \le ... \le u_{i+k+1}$, and it is the basic spline in polynomial spline space. This is why it was named as B-spline.

2.2.3 The choice of B-spline parameters

Usually, control points and degree are the main considerable factors in the choice of B-spline parameters.It can adjust the shape of the B-spline by changing the control points and degree.

According to the former researchers[9], when increasing the degree, the B-spline will become more and more smooth. But at the same time, there will have higher peak value in the velocity curve and acceleration curve. So it should choose a proper degree of the B-spline. And by changing the control points, it can improve the relationship between the B-spline curve and control polygon. The more the control points choose, the better the B-spline curve shows.

III. RESULTS

In this paper, it uses the software MATLAB to reconstruct the follower motion curve. One of the advantages of MATLAB is drawing the curve through a series of instructions. By help with this function, it can easily accomplish the reconstruction of the follower motion curve [10].

In order to get the follower motion curve that has smooth and continuous acceleration part, this paper chooses the degree of B-spline equal to 3. Meanwhile, the motion curve should keep the velocity part and acceleration part approximating 0 at the start and the end so that there will not result in jerking motion, it chooses 46 control points.

According to the formula of the B-spline, it can get the data points from the mathematical equation of 3 different curves firstly, then using the formula to calculate the knot vector and control points. The amount of control points is chosen as 51. At last, it can reconstruct the follower motion curve by the instructions. The results are shown below:

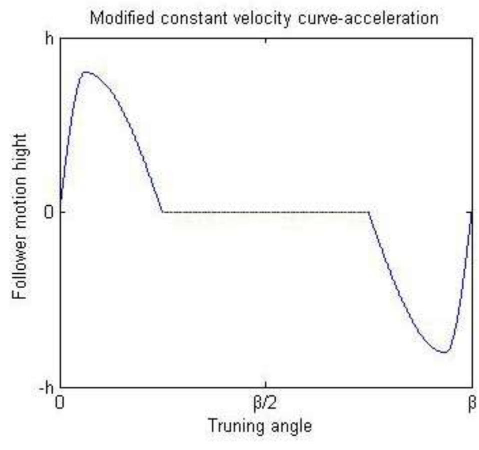

Fig. 4 the B-spline construction of modified constant velocity curve

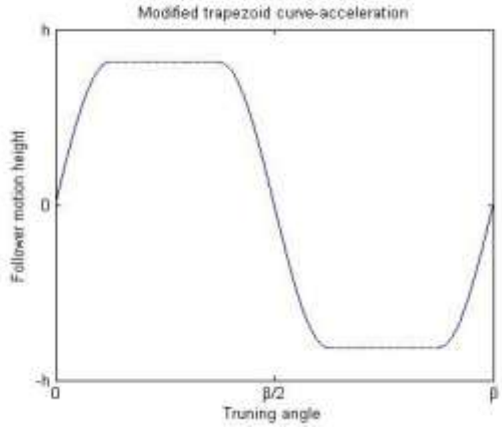

Fig. 5the B-spline construction of modified trapezoid curve

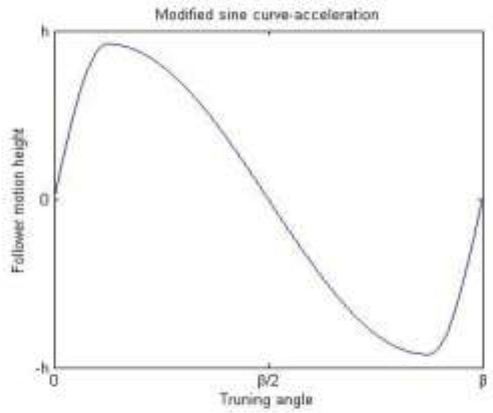

Fig. 6the B-spline construction of modified sine curve

IV. CONCLUSIONS

Compared with the original follower motion acceleration curve, it can be observed that the B-spline reconstruction of the follower is similar to the original curve. It proved that reconstruct the follower motion curve for the non-development surface by B-spline is feasible. In practical application, B-spline fitting in the design stage is well matched with B-spline interpolation in the manufacturing stage. This method can reduce the design error in the entire process.

However, there still have some limitations in this paper. The disadvantage of the B-spline is the same weights of all data points. But in actual situation, weights in every data point is different. So there will exist some theoretical error. So, in the future research, it can try to use NURBS(Non-uniform rational B-spline) to reconstruct the follower motion curve.

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